EXPERIMENTAL STUDY OF THE ORIGIN OF TURBULENCE IN NON-NEWTONIAN LIQUID FLOWS

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A description is given of an experimental apparatus and method for determining flow crisis in 1, 2, and 3% solutions of polyvinyl alcohol in water. The results of the experiment are compared with calculations.

We will consider the motion of liquids with structural viscosity obeying the rheological equation [1]

$$\frac{\varphi-\varphi_\infty}{\varphi_0-\varphi_\infty}=\exp\left(-\Theta\,\frac{\tau}{\varphi_\infty-\varphi_0}\right). \qquad (1)$$

The steady-state isothermal flow of these liquids is determined by the following system of parameters (to be specific we will consider motion in a circular tube):

$$\rho,\ \varphi_0,\ \varphi_\infty,\ \Theta,\ D,\ \bar{w}. \qquad (2)$$

This system differs from the particular case of a Newtonian liquid in that instead of the viscosity it contains the three parameters φ_0, φ_∞ and Θ. Since the number of basic dimensions does not change, in accordance with the pi-theorem our case will be characterized not by one criterion—the Reynolds number—but by three dimensionless quantities composed of the parameters of system (2), for example,

$$\mathrm{Re}_0=\rho\varphi_0 D\bar{w},$$
$$\Theta_*=\rho\varphi_0^3 D^2/|\Theta|,$$
$$\alpha_*=\varphi_\infty/\varphi_0. \qquad (3)$$

Solving the equations of motion and continuity for the laminar flow regime of a liquid governed by rheological equation (1), we find the relation between the dimensionless parameters in the form

$$\frac{\alpha_* A}{4}+(\alpha_*-1)A\left[a\exp(-B)-\frac{6}{B^4}\right]+1=0. \qquad (4)$$

Here,

$$A=\frac{\zeta\,\mathrm{Re}_0}{16},\quad B=\frac{\zeta\,\mathrm{Re}_0^2}{8\Theta_*(\alpha_*-1)},$$
$$a=\frac{1}{B}+\frac{3}{B^2}+\frac{6}{B^3}+\frac{6}{B^4}.$$

Equation (4) contains, apart from the parameters (3), an unknown quantity, the resistance coefficient ζ, which, generally speaking, is a function of these parameters.

We will use the constancy of the resistance coefficient at the onset of the transition regime and obtain in explicit form the relation between the critical Reynolds number Re_* and the parameters Θ_* and α_*.

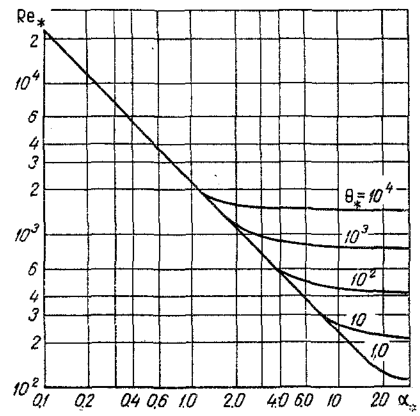

Fig. 1. Critical Reynolds number Re_* as a function of the numbers Θ_* and α_* (calculated from Eq. (4)).

Equation (4) was solved by Newton's method on an M-20 computer. The results are shown in Fig. 1. They are valid for liquids that do not possess elastic properties.

In the experimental investigation of media with structural viscosity it is important to measure directly the rheological characteristics of the liquid investigated.

The relation between fluidity φ and shear stress was measured with a set of capillary viscometers. We investigated aqueous solutions of polyvinyl alcohol (PVA) at concentrations of 1, 2, and 3%. The flow curves (Fig. 2) were recorded at the beginning and end of the experiment and were found to be stable.

In order to estimate the elastic properties of the liquid we set up a parallel auxiliary loop, in which the normal stress difference was measured. From a constant-level tank 1 (Fig. 3) the liquid entered a plexiglas channel 2 measuring $10.2 \times 42.5 \times 1100$ mm. At a distance of 850 mm from the channel inlet we inserted a pitot tube made from a hypodermic needle 0.8 mm in diameter. A hole 0.8 mm in diameter was drilled in the channel opposite the end of the tube.

In the same cross-section we recorded the velocity profiles with the optico-mechanical instrument described in [2].

The static pressure probe 4 and the pitot tube 5 were connected with a ∩-shaped two-liquid differential manometer 6 (the second liquid was chlorani-

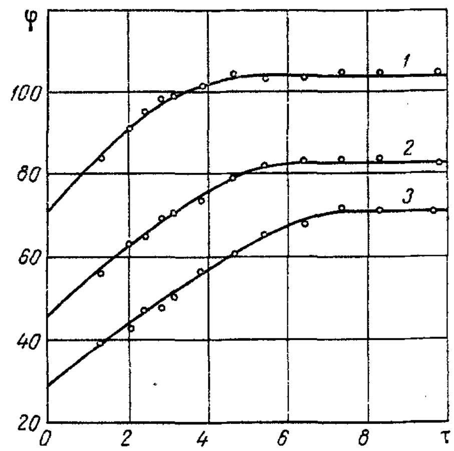

Fig. 2. Fluidity φ, $m^2/N \cdot sec$, as a function of the shear stress τ, N/m^2, for 1, 2, and 3% aqueous solutions of PVA (1, 2, 3, respectively).

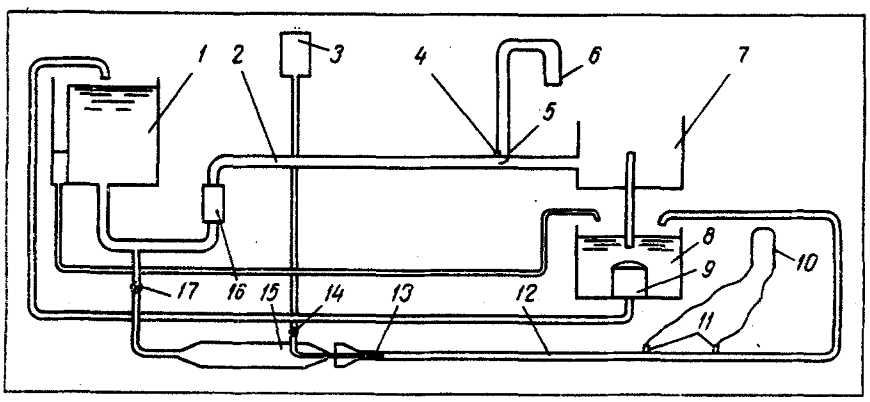

Fig. 3. Experimental setup: 1) constant-level tank, 2) rectangular duct, 3) dye tank, 4, 11) static pressure probes, 5) pitot tube, 6) differential U manometer, 7) overflow tank, 8) tank, 9) electric motor, 10) differential Ո manometer, 12) working tube, 13) dye capillary, 14, 17) valves, 15) glass tube, 16) rotameter.

line, ϱ = 1216 kg/m^3 at 20°C). The manometer readings were obtained with a KM-6 cathetometer.

In order to determine the normal stress difference in the laminar flow regime we deducted from the readings of manometer 6 the velocity head determined from the velocity profile recorded with the optico-mechanical instrument.

A similar method was described in [3]. In [3], however, it was proposed to calculate the velocity head on the basis of the Ostwald-De Waele power law and not to measure it directly. The exponent was assumed constant whereas in reality it depends on the shear stress. Since the shear stress varies over the channel cross-section, this method may lead to a serious deviation from the true velocity profile.

The experiment showed that the velocity heads measured with a pitot tube are in agreement with the velocity profiles measured by the optico-mechanical method. This enabled us to conclude that in our case there was practically no normal-stress difference.

Laminar flow crisis was determined by two independent methods: 1) from the effect on a jet of dye, 2) from the break in the pressure drop-flow rate curve.

From the constant-level tank 1 the liquid entered a glass tube 15 with an inside diameter of 40 mm and a length of 500 mm. Tube 15 was connected with the working tube 12 by means of a standard 29 × 24 mm ground-glass joint. Four working tubes with inside diameters of 5, 6, 8 and 10 mm were employed. Into tube 15 we sealed a capillary 13, through which a jet of dye (filtered solution of brilliant green) entered the working tube from tank 3. The rate of flow of dye was regulated by means of valve 14. The static pressure was registered through holes 0.8 mm in diameter. The distance between static holes 11 was equal to 100 mm. The first hole was 50 diameters from the inlet. The static holes were connected to an inverted-U differential manometer 10. From the working tube the liquid entered tank 8 from which it was returned to tank 1 by pump 9. The liquid flow rate could be varied by means of valve 17. The flow rate was measured volumetrically

The experiments were performed as follows. By gradually opening valve 17, we determined the relation between pressure drop and flow rate and registered the moment of washout of the dye. We then plotted curves relating the shear stress with the flow rate and the resistance coefficient with Re_0. The break in the curves corresponds to the onset of the transition regime. The shear stresses at the wall at the flow crisis points considerably exceeded the values of τ corresponding to a linear flow law.

The discrepancy in the values of Re_* obtained by the two methods mentioned above did not exceed 4%. A check with distilled water gave a value $Re_* \approx 2300$.

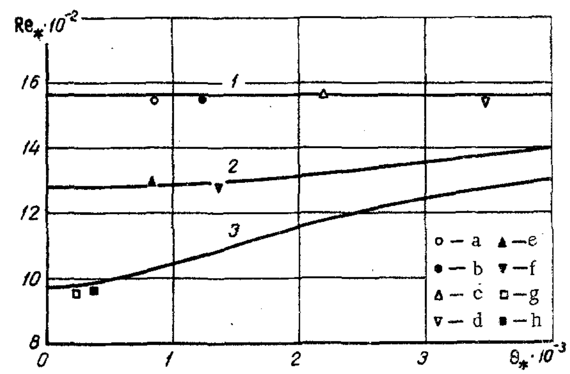

Fig. 4. Critical Reynolds number Re_* as a function of Θ_* and α_*: 1, 2, 3) calculated from (4) with α_* = 1.47, 1.80 and 2.36; a,b,c,d) experiments with 1% solutions and working tube diameters of 5, 6, 8, and 10 mm; e, f) experiment with 2% solutions and diameters of 8 and 10 mm; g, h) the same for 3% solutions.

It is clear from Fig. 4 that the agreement between experiment and calculation may be described as satisfactory.

NOTATION

φ is the fluidity; Θ is the coefficient stability; τ is the modulus of shear stress; φ_0 and φ_∞ are the limiting values of fluidity at zero and infinitely large shears, respectively; ρ is the density of liquid; D is the diameter of tube; $\bar{w}$ is the mean velocity; ξ is the resistance coefficient; Re_* is the critical value of the number Re_0.

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